CHAPTER 3 – INVENTORY MANAGEMENT

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- Inventory models
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DEFINITION AND TYPES OF INVENTORY

- Inventory is supplies of goods and materials (incl. raw materials, work in progress, and finished goods) that are held by an organization.
- Types of inventory
  - Cycle stock
  - Safety stock
  - In-transit stock
  - Speculative stock

PURPOSES OF INVENTORY

- Be a buffer between different parts of the supply chain
- Allow for demands that are larger than expected or unexpected
- Allow for deliveries that are delayed
- Take advantage of price discounts on large orders
- Allow the purchase of items when the price is low
- Allow the purchase of scarce items
- Allow for seasonal operations
- Make full loads and reduce transport costs
- Cover for emergencies
- Be profitable when inflation is high

INVENTORY COSTS

- Carrying (holding) costs
  - Obsolescence costs
  - Inventory shrinkage
  - Storage costs
  - Handling costs
  - Insurance, tax, interest
- Ordering costs
- Stock-out (shortage) costs

INVENTORY MODELS

- Single-period models: based on one-time purchasing decision
- Fixed-order quantity models (continuous system): inventory is continuously tracked and an order is placed when the inventory declines to a reorder point
- Fixed-time period models (periodic system): inventory is checked at regular periodic intervals and an order is placed to raise the inventory level to a specified threshold

FIXED-ORDER QUANTITY MODEL – EOQ MODEL

Assumptions

- *A continuous, constant, and known rate of demand*
- *A constant and known lead time*
- *A constant purchase price, independent of the order quantity*
- *All demand is satisfied*
- *Inventory holding cost is based on average inventory*
- *A constant ordering cost*

→ Determine order quantity that the total cost (incl. purchase cost, ordering cost, holding cost) is minimized

FIXED-ORDER QUANTITY MODEL – EOQ MODEL

$$TC = DC + \frac{D}{Q}S + \frac{Q}{2}H$$

$TC: Total\ annual\ cost$

$D: Annual\ demand$

$C: Cost\ per\ unit$

$Q: Order\ quantity$

$S: Ordering\ cost$

$H: Annual\ holding\ cost, hC$

$$\Rightarrow \quad Q^* = \sqrt{\frac{2DS}{hC}}$$

EOQ MODEL - EXAMPLE

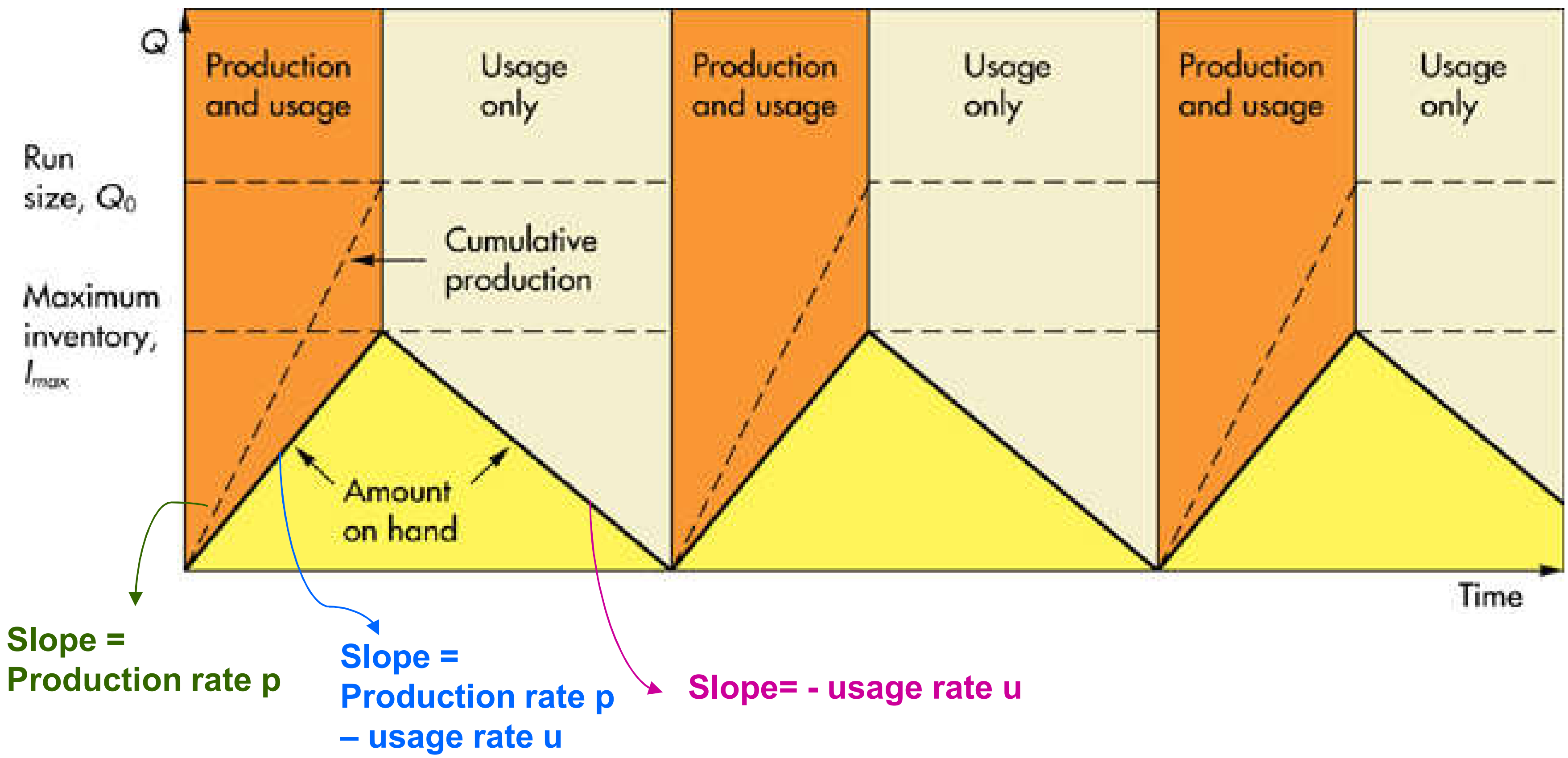
Demand for a smartphone at a store is 75 units per month. The store incurs a fixed order cost of $700 each time an order is placed. Each smartphone costs the store $800 and the store has an annual holding cost of 20 percent per unit. What is the optimal order size in each replenishment?

EPQ MODEL

Assumptions

- *Only one product involved*
- *Annual demand is known*
- *The usage rate is constant*
- *Usage occurs continually*
- *Production occurs periodically*
- *The production rate is constant*
- *Lead time is known and constant*
- *There are no quality discounts*

EPQ MODEL


Q
Run size, Q_0
Maximum inventory, I_{max}
Production and usage
Usage only
Production and usage
Usage only
Production and usage
Usage only
Cumulative production
Amount on hand
Time
Slope = Production rate p
Slope = Production rate p – usage rate u
Slope= - usage rate u

EPQ MODEL – ECONOMIC RUN QUANTITY

$$Q_p^* = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}}$$

$D: Annual\ demand$

$S: Setup\ cost$

$H: Holding\ cost\ per\ unit\ per\ year$

$p: Production\ or\ delivery\ rate$

$u: Usage\ rate$

EPQ MODEL - EXAMPLE

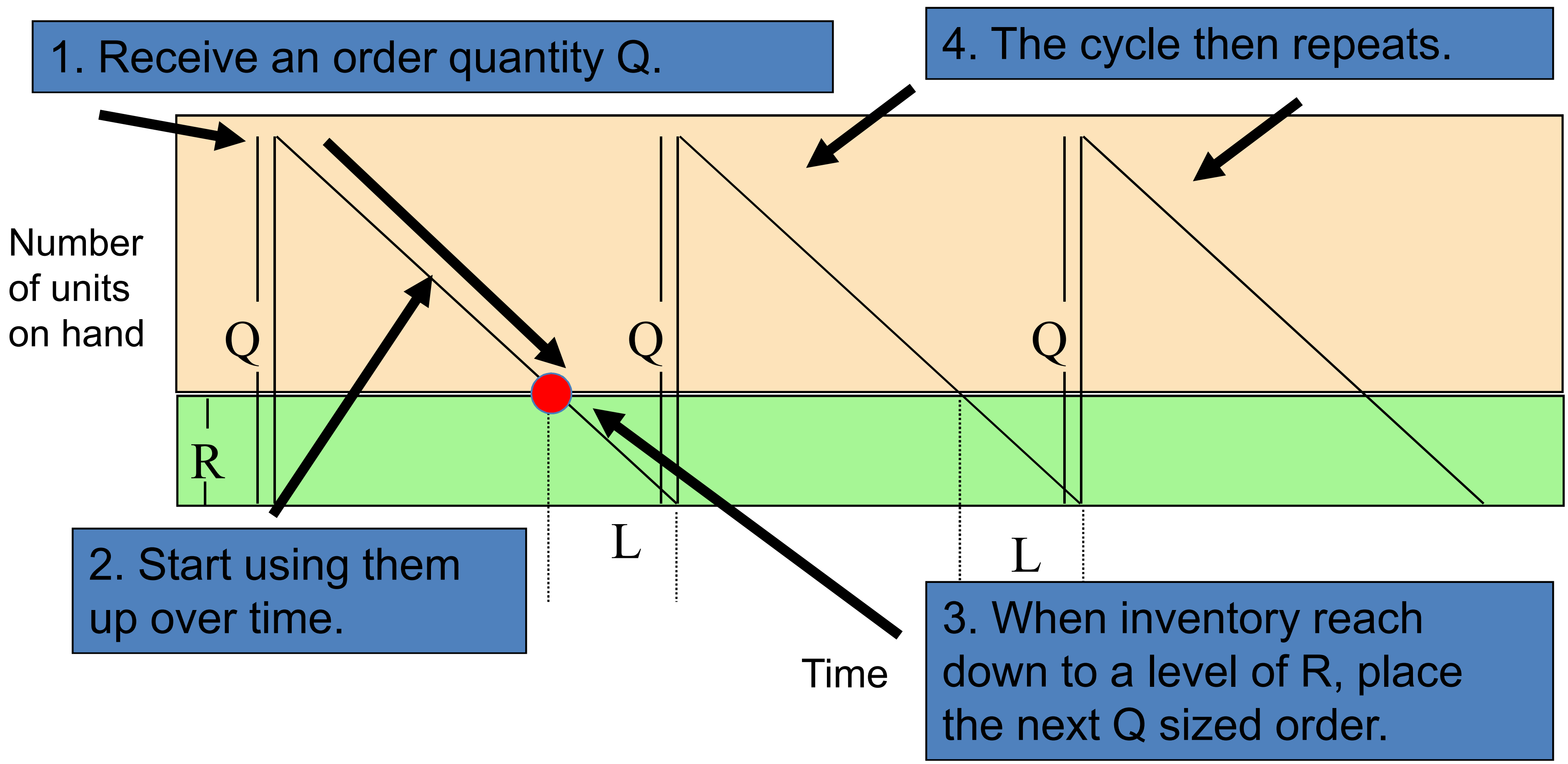
A toy manufacturer uses 48,000 rubber wheels per year for its popular toy car series. The firm makes its own wheels, which it can produce at a rate 800 per day. The toy cars are assembled uniformly over entire year. Holding cost is $1 per wheel a year. Setup costs for a production run of wheels is $45. The firm operates 240 days per year. What is the optimal run size?

LITTLE'S LAW

The average amount of inventory in a system is proportional to the time it takes for inventory to flow through the system

$$\mathrm{I} = D \times T$$

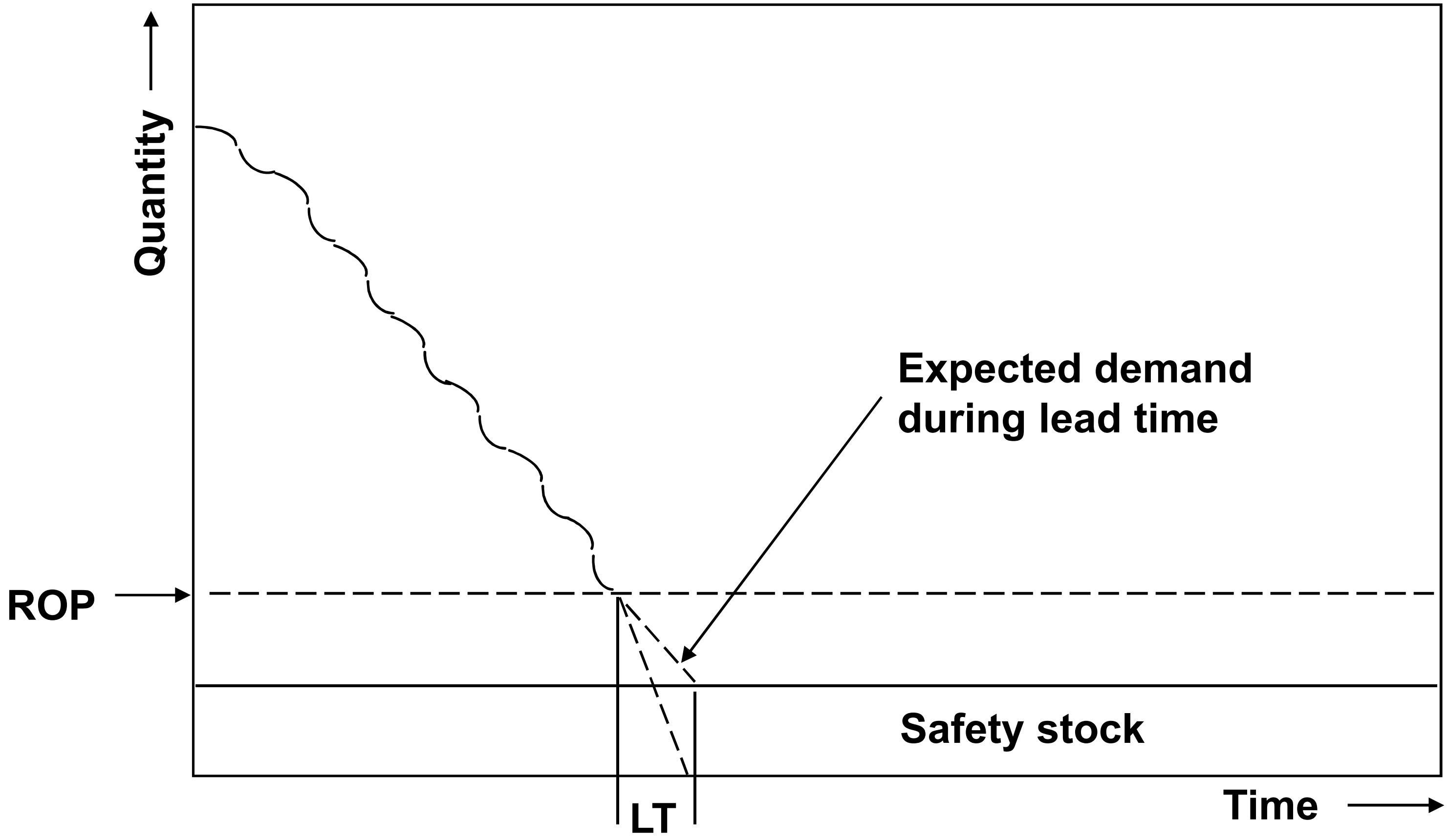
REORDER POINT



$$Certainty\ (demand\ \&\ lead\ time):\ ROP = D \times L$$

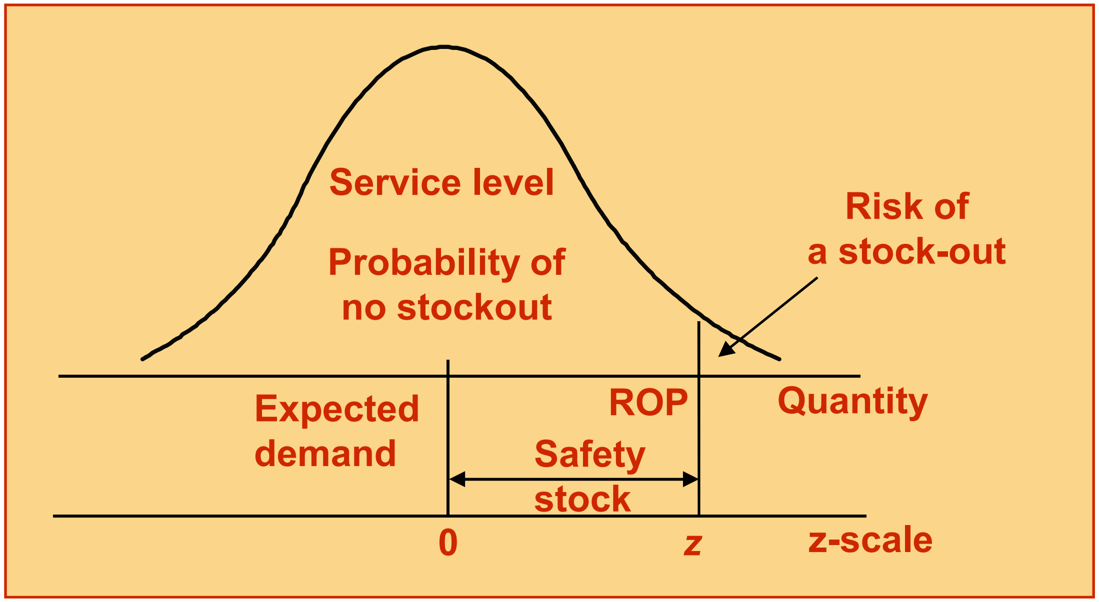
SAFETY STOCK

The amount of inventory carried in addition to the expected demand to reduce risk of stock-out during lead time



SERVICE LEVEL

The probability of not having a stock-out in a replenishment cycle



$$When\ only\ demand\ is\ uncertain:\ ROP = D_L + ss$$

SAFETY STOCK

$$ROP = D_L + ss$$

$$Expected\ demand\ during\ lead\ time:\ D_L = D \times L$$

$$Safety\ stock:\ ss = z \times \sigma_L$$

$$z: number\ of\ standard\ deviation\ for\ a\ specified\ service\ level$$

$$Standard\ deviation\ of\ demand\ during\ lead\ time:\ \sigma_L = \sqrt{L}\sigma_D$$

STANDARD NORMAL DISTRIBUTION

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

EXERCISE

Weekly demand for coke at a supermarket is normally distributed, with a mean of 1,500 bottles and a standard deviation of 300. The replenishment lead time is two weeks. Assuming that a continuous review policy is used, please calculate the safety stock that the supermarket should carry to achieve a service level of 90 percent. What is the reorder point?

FIXED-TIME PERIOD MODEL

$$Order - Up - to\ Level:\ OUL = D_{T+L} + ss$$

$$Mean\ demand\ during\ T + L\ periods, D_{T+L} = (T + L)D$$

$$Standard\ deviation\ of\ demand\ during\ T + L\ periods, \sigma_{T+L} = \sqrt{T + L}\sigma_D$$

$$Safety\ stock:\ ss = z \times \sigma_{T+L}$$

$$Order\ quantity = OUL - Current\ inventory\ on\ hand$$

Weekly demand for coke at a supermarket is normally distributed, with a mean of 1,500 bottles and a standard deviation of 300. The replenishment lead time is two weeks. Assuming that an inventory review of every three weeks is conducted, please calculate the safety stock that the supermarket should carry to achieve a service level of 90 percent. What is the order-up-to level?

SINGLE-PERIOD MODEL

C_o: $Cost\ of\ overstocking\ by\ one\ unit, C_o = c - s$

C_u: $Cost\ of\ understocking\ by\ one\ unit, C_u = p - c$

p: $retail\ price\ per\ unit$

c: $unit\ cost$

s: $salvage\ value\ per\ unit$

P: $the\ probability\ that\ demand\ will\ be\ at\ or\ below\ Q^*$

$$P\ (demand \leq\ Q^*) = \frac{C_u}{C_u +\ C_o}$$

$$Optimal\ order\ size:\ Q^* =\ \overline{D} + z\ \times \sigma_D$$

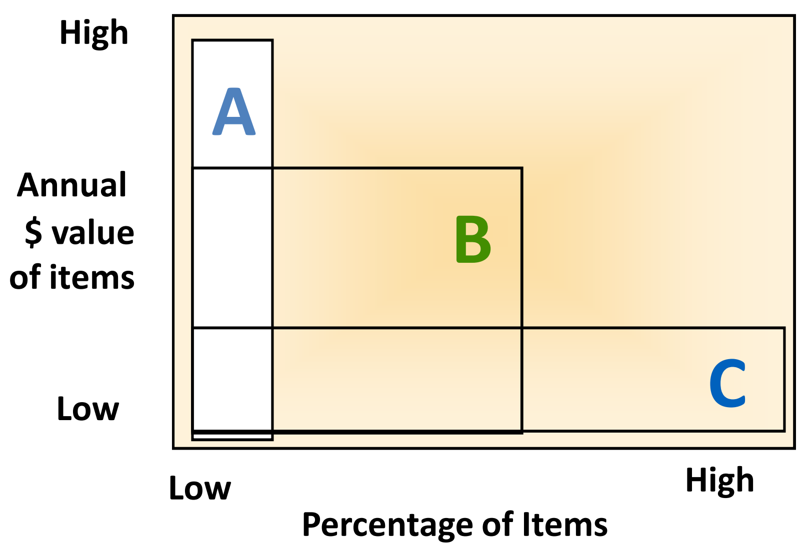
EXERCISE

A school organize a tournament game this month. Based on the past experience the tournament organizer sells on average 1,500 T-shirts with a standard deviation of 200. We make $10 on every shirt we sell at the game, but lose $5 on every shirt not sold. How many shirts should we make for the game?

ABC ANALYSIS OF INVENTORY

ABC inventory classification is to classify items into groups to establish the appropriate contrive over each item.

- A items: 20% of items (making up 80% of annual dollar usage)
- B items: 40% of items (making up 15% of annual dollar usage)
- C items: 40% of items (making up 5% of annual dollar usage)



ABC INVENTORY CLASSIFICATION

Part Number	Annual Unit Usage	Unit Cost ($)	Annual $ Usage	Annual $ Usage (%)
1	1,100	2	2,200	6%
2	600	40	24,000	63%
3	100	4	400	1%
4	1,300	1	1,300	3%
5	100	60	6,000	16%
6	10	25	250	1%
7	100	2	200	1%
8	1,500	2	3,000	8%
9	200	2	400	1%
10	500	1	500	1%
			38,250	

ABC INVENTORY CLASSIFICATION

Part Number	Annual $ Usage	Annual $ Usage (%)	Cummulative $ Usage	Cummulative % $ Usage	Cummulative % of Items
2	24,000	63%	24,000	63%	10%
5	6,000	16%	30,000	78%	20%
8	3,000	8%	33,000	86%	30%
1	2,200	6%	35,200	92%	40%
4	1,300	3%	36,500	95%	50%
10	500	1%	37,000	97%	60%
3	400	1%	37,400	98%	70%
9	400	1%	37,800	99%	80%
6	250	1%	38,050	99%	90%
7	200	1%	38,250	100%	100%

Classification	Percentage of Items	Percentage of $ Usage	Value per Class
A	20%	78%	30,000
B	40%	18%	7,000
C	40%	3%	1,250
Total	100%	100%	

INVENTORY TURNOVER

The number of times that inventory is sold in a one-year period

$$\frac{Cost\ of\ Goods\ Sold}{Average\ Inventory}$$

The higher ratio → the better performance!

VENDOR MANAGED INVENTORY (VMI)

VMI is an approach in which a supplier is responsible for managing and replenishing inventory.

Steps

- Agree a contract
- Share information
- Monitor the process
- Replenish inventory
- Payment

SAFETY STOCK IN CENTRALIZED VS DECENTRALIZED SYSTEMS

$D_i: Mean\ periodic\ demand\ in\ region\ i, i = 1, \ldots, k$

$\sigma_i: Standard\ deviation\ of\ periodic\ demand\ in\ region\ i, i = 1, \ldots, k$

$\rho_{ij}: Correlation\ of\ periodic\ demand\ for\ regions\ i, j, 1 \leq i \neq j \leq k$

DECENTRALIZED OPTION

$$Total\ safety\ inventory\ in\ decentralized\ option = \sum_{i=1}^{k} z \times \sigma_{L_i} = \sum_{i=1}^{k} z \times \sqrt{L} \times \sigma_i$$

$\sigma_{L_i}: Standard\ deviation\ of\ demand\ during\ lead\ time\ in\ region\ i$

SAFETY STOCK IN CENTRALIZED VS DECENTRALIZED SYSTEMS

CENTRALIZED OPTION

$\sigma_i: Standard\ deviation\ of\ demand\ in\ region$ i

$\sigma_j: Standard\ deviation\ of\ demand\ in\ region$ j

Mean demand at the centralized region: $D^C = \sum_{i=1}^{k} D_i$

$Variance\ of\ demand\ at\ the\ centralzied\ region:\ var(D^C)$

$$= \sum_{i=1}^{k} \sigma_i^2 + 2\sum_{i>j} \rho_{ij}\sigma_i\sigma_j$$

$$\sigma_D^C = \sqrt{var(D^C)}$$

$$Safety\ inventory\ on\ aggregation = z \times \sqrt{L} \times \sigma_D^C$$

CENTRALIZED OPTION

$If\ all\ k\ regions\ have\ demand\ that\ is\ independent, \rho_{ij} = 0$

$Assumption: Demand\ at\ each\ region\ is\ the\ same$

$$D^C = kD$$

$$\sigma_D^C = \sqrt{k}\sigma_D$$

EXERCISE

A car dealership has four outlets serving the entire and large province. Weekly demand at each outlet is normally distributed, with a mean of 25 cars and a standard deviation of 5. The lead time for replenishment from the manufacturer is 2 weeks. Each outlet covers a separate geographic area, and the demand across any pair of areas is independent. The dealership is considering the possibility of replacing the four outlets with a single large outlet. Assume that the demand in the central outlet is the sum of the demand across all four areas. The dealership is targeting a service level of 0.90. Compare the level of safety inventory needed in the two options.

RISK POOLING

Centralized option (aggregation) reduces the required safety inventory — as long as the demand being aggregated is not perfectly positively correlated ($\rho=1$).

Risk pooling: Demand variability is reduced if demand is aggregated demand across locations → safety stock is reduced.

CHAPTER 3 – INVENTORY MANAGEMENT

THANK YOU!